Strange axial-vector mesons mixing angle

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Abstract

The masses of the $K_1(^3P_1)$ and $K_1(^1P_1)$ are considered in a nonrelativistic constituent quark model, and the absolute value of the $K_1(^3P_1) - K_1(^1P_1)$ mixing angle is determined to be about $59.29°$. Comparison of the theoretical predictions on the strong decay widths of the $K_1(1270)$ and $K_1(1400)$ in the $^3P_0$ decay model as well as the production ratio of these two states in the τ decay between the available experimental data strongly favors that the $K_1(^3P_1) - K_1(^1P_1)$ mixing angle is about $+59.29°$.

Key words: Other strange mesons; Nonrelativistic quark model; Hadronic decays of mesons; Decays of taus

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1 Introduction

The strange axial vector mesons provide interesting possibilities to study the QCD in the nonperturbative regime by the mixing of the $^3P_1$ and $^1P_1$ states. In the exact SU(3) limit, the $K_1(^3P_1)$ and $K_1(^1P_1)$ do not mix, just as the a_1 and b_1 mesons do not mix. For the strange quark mass greater than the up and down quark masses so that SU(3) is broken, also, the $K_1(^3P_1)$ and $K_1(^1P_1)$ do not possess definite C-parity, therefore these states can in principle mix to give the physical $K_1(1270)$ and $K_1(1400)$.

Accurate determination of θ_K, the mixing angle of the $K_1(^3P_1)$ and $K_1(^1P_1)$, is important for comparing the theory predictions about the decays involving the strange axial-mesons with the experimental data. In the literature, θ_K has been estimated by some different approaches, however, there is not yet a consensus on the value of θ_K. As the optimum fit to the data as of 1977, Carnegie et al. finds $\theta_K = (41 \pm 4)^\circ$[1]. Within the heavy quark effective theory Isgur and Wise predict two possible mixing angles, $\theta_K \sim 35.3^\circ$ and $\theta_K \sim -54.7^\circ$[2]. Based on the analysis of $\tau \to \nu K_1(1270))$ and $\tau \to \nu K_1(1400))$, Rosner suggests $\theta_K \sim 62^\circ$[3], Asner et al. gives $\theta_K = (69 \pm 16 \pm 19)^\circ$ or $(49 \pm 16 \pm 19)^\circ$[4], and Cheng obtains $\theta_K = \pm 37^\circ$ or $\pm 58^\circ$[5]. From the experimental information on masses and the partial rates of $K_1(1270)$ and $K_1(1400)$, Suzuki finds two possible solutions with a two-fold ambiguity, $\theta_K \sim 33^\circ$ or 57°[6]. A constraint $35^\circ \leq \theta_K \leq 55^\circ$ is predicted by Burakovsky et al. in a nonrelativistic constituent quark model[7], and within the same model, the values of $\theta_K \simeq (31 \pm 4)^\circ$ and $\theta_K \simeq (37.3 \pm 3.2)^\circ$ are suggested by Chliapnikov[8] and Burakovsky[9], respectively. The calculations for the strong decays of $K_1(1270)$ and $K_1(1400)$ in the $^3P_0$ decay model suggest $\theta_K \sim 45^\circ$[10, 11]. The mixing angles $\theta_K \sim 34^\circ$[12], $\theta_K \sim 5^\circ$[13] are also presented within a relativized quark model. Vijande et al. suggests $\theta_K \sim 55.7^\circ$ based on the calculations in a constituent quark model[14]. More recently, based on the $f_1(1285) - f_1(1420)$ mixing angle $\sim 50^\circ$ derived from the analysis for a substantial body of data concerning the $f_1(1420)$ and $f_1(1285)$[15], we suggest that the $K_1(^3P_1) - K_1(^1P_1)$ mixing angle is about $\pm(59.55 \pm 2.81)^\circ$[16].

In the present work, we shall show that the $K_1(^3P_1) - K_1(^1P_1)$ mixing angle derived from the nonrelativistic constituent quark model is in good agreement with that given by Ref.[16], and try to constrain the sign of the $K_1(^3P_1) - K_1(^1P_1)$ mixing angle by considering the open-flavor strong decays of the $K_1(1270)$ and $K_1(1400)$ in the $^3P_0$ decay model and the production ratio of these two states in the τ decay.

2 Nonrelativistic constituent quark model for P-wave mesons

In the constituent quark model, the conventional $q\bar{q}$ wave function is typically assumed to be a solution of a nonrelativistic Schrödinger equation with the generalized Breit-Fermi Hamiltonian which contains a QCD inspired potential $V(\mathbf{r})$[17]. The phenomenological forms of the matrix element of the Breit-Fermi Hamiltonian for the $q\bar{q}$ mesons with orbital angular momentum L are given by[8, 18]:

$$M_{L=0} = m_q + m_{\bar{q}} + e_0 \frac{\langle \mathbf{s}_q \cdot \mathbf{s}_{\bar{q}} \rangle}{m_q m_{\bar{q}}}, \tag{1}$$

$$\begin{aligned} M_{L\neq 0} &= m_q + m_{\bar{q}} + a_L + b_L \left(\frac{1}{m_q} + \frac{1}{m_{\bar{q}}} \right) + c_L \left(\frac{1}{m_q^2} + \frac{1}{m_{\bar{q}}^2} \right) + \frac{d_L}{m_q m_{\bar{q}}} + e_L \frac{\langle \mathbf{s}_q \cdot \mathbf{s}_{\bar{q}} \rangle}{m_q m_{\bar{q}}} \\ &+ f_L \left(\frac{1}{m_q^3} + \frac{1}{m_{\bar{q}}^3} \right) + g_L \left[\frac{(m_q + m_{\bar{q}})^2 + 2 m_q m_{\bar{q}}}{4 m_q^2 m_{\bar{q}}^2} \langle \mathbf{L} \cdot \mathbf{S} \rangle - \frac{m_q^2 - m_{\bar{q}}^2}{4 m_q^2 m_{\bar{q}}^2} \langle \mathbf{L} \cdot \mathbf{S}_- \rangle \right] \\ &+ \frac{h_L}{m_q m_{\bar{q}}} \langle \mathbf{S}_{q\bar{q}} \rangle, \end{aligned} \tag{2}$$

where m_q and $m_{\bar{q}}$ are the constituent quark masses, $\mathbf{s}_q$ and $\mathbf{s}_{\bar{q}}$ are the constituent quark spins, e_0, a_L, b_L, c_L, d_L, e_L, f_L, g_L and h_L are constants, $\mathbf{S} = \mathbf{s}_q + \mathbf{s}_{\bar{q}}$, $\mathbf{S}_- = \mathbf{s}_q - \mathbf{s}_{\bar{q}}$, and $\mathbf{S}_{q\bar{q}} = 3\frac{(\mathbf{s}_q \cdot \mathbf{r})(\mathbf{s}_{\bar{q}} \cdot \mathbf{r})}{r^2} - \mathbf{s}_q \cdot \mathbf{s}_{\bar{q}}$. Angular momentum part of the matrix elements of (1) and (2) is shown in Table 1.

	$^3P_2$	$^3P_1$	$^3P_0$	$^1P_1$	$^3S_1$	$^1S_0$
$\langle \mathbf{s}_q \cdot \mathbf{s}_{\bar{q}} \rangle$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$
$\langle \mathbf{L} \cdot \mathbf{S} \rangle$	1	-1	-2	0		
$\langle \mathbf{S}_{q\bar{q}} \rangle$	$-\frac{2}{5}$	2	-4	0		
$\langle \mathbf{L} \cdot \mathbf{S}_- \rangle$	0	0	0	$\frac{3}{2}$		

Table 1: Angular momentum part of the matrix elements of (1) and (2).

With the help of Table 1, applying (1) and (2) to S-wave and P-wave mesons, in the SU(2) flavor symmetry limit, one can obtain[1]

$$\frac{M_\pi + 3M_\rho}{2M_K + 6M_{K^*} - M_\pi - 3M_\rho} = \frac{m_u}{m_s} = 0.6298 \pm 0.00068, \tag{3}$$

and

$$\frac{M(^3P_2)_{s\bar{s}} - M(^1P_1)_{s\bar{s}}}{M(^3P_2)_{n\bar{n}} - M(^1P_1)_{n\bar{n}}} = \frac{m_u^2}{m_s^2}. \tag{4}$$

From (4), with the help of the Gell-Mann-Okubo mass formula[20]

$$M^2(^3P_2)_{s\bar{s}} + M^2(^3P_2)_{n\bar{n}} = 2M^2_{K(^3P_2)}, \tag{5}$$

$$M^2(^1P_1)_{s\bar{s}} + M^2(^1P_1)_{n\bar{n}} = 2M^2_{K_1(^1P_1)}, \tag{6}$$

taking $M(^3P_2)_{n\bar{n}} = M_{a_2(1320)} = 1318.3 \pm 0.6$ MeV, $M(^1P_1)_{n\bar{n}} = M_{b_1(1235)} = 1229.5 \pm 3.2$ MeV and $M_{K(^3P_2)} = M_{K_2^*(1430)} = 1429 \pm 0.99$ MeV , one can arrive that

$$M_{K(^1P_1)} = 1369.52 \pm 1.92 \text{ MeV}. \tag{7}$$

The $K_1(^3P_1)$ and $K_1(^1P_1)$ can mix to produce the physical states $K_1(1400)$ and $K_1(1270)$ and the mixing between $K_1(^3P_1)$ and $K_1(^1P_1)$ can be parameterized as[6]

$$\begin{aligned} K_1(1400) = &\quad K_1(^3P_1)\cos\theta_K - K_1(^1P_1)\sin\theta_K, \\ K_1(1270) = &\quad K_1(^3P_1)\sin\theta_K + K_1(^1P_1)\cos\theta_K, \end{aligned} \tag{8}$$

where θ_K denotes the $K_1(^3P_1) - K_1(^1P_1)$ mixing angle. Without any assumption about the origin of the $K_1(^3P_1) - K_1(^1P_1)$ mixing, the masses of the $K_1(^3P_1)$ and $K_1(^1P_1)$ can be related to $M_{K_1(1400)}$ and $M_{K_1(1270)}$, the masses of the $K_1(1400)$ and $K_1(1270)$, by the following relation phenomenologically,

$$S \begin{pmatrix} M^2_{K_1(^3P_1)} & A \\ A & M^2_{K_1(^1P_1)} \end{pmatrix} S^\dagger = \begin{pmatrix} M^2_{K_1(1400)} & 0 \\ 0 & M^2_{K_1(1270)} \end{pmatrix}, \tag{9}$$

where A denotes a parameter describing the $K_1(^3P_1) - K_1(^1P_1)$ mixing , and

$$S = \begin{pmatrix} \cos\theta_K & -\sin\theta_K \\ \sin\theta_K & \cos\theta_K \end{pmatrix}.$$

[1]Where $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$, All the masses used as input in the present work are taken from PDG[19].

From (9), one can have

$$M^2_{K_1(^3P_1)} = M^2_{K_1(1400)}\cos^2\theta_K + M^2_{K_1(1270)}\sin^2\theta_K, \tag{10}$$

$$M^2_{K_1(^1P_1)} = M^2_{K_1(1400)}\sin^2\theta_K + M^2_{K_1(1270)}\cos^2\theta_K, \tag{11}$$

$$\cos(2\theta_K) = \frac{M^2_{K_1(^3P_1)} - M^2_{K_1(^1P_1)}}{M^2_{K_1(1400)} - M^2_{K_1(1270)}}. \tag{12}$$

Inputting $M_{K_1(1400)} = 1402 \pm 7$ MeV, $M_{K_1(1270)} = 1273 \pm 7$ MeV and $M_{K_1(^3P_1)} \simeq 1369.52 \pm 1.92$ MeV shown in (7), from (10)-(12), we have

$$M_{K_1(^3P_1)} = 1307.88 \pm 10.33 \text{ MeV}, \quad \theta_K = \pm(59.29 \pm 2.87)^\circ. \tag{13}$$

Obviously, the present result that $(M_{K_1(^1P_1)}, M_{K_1(^3P_1)}) = (1369.5 \pm 1.92, 1307.88 \pm 10.33)$ MeV and $\theta_K = \pm(59.29 \pm 2.87)^\circ$ is in good agreement with that $(M_{K_1(^1P_1)}, M_{K_1(^3P_1)}) = (1370.03 \pm 9.69, 1307.35 \pm 0.63)$ MeV and $\theta_K = \pm(59.55 \pm 2.81)^\circ$ given by Ref.[16] based on the $f_1(1285) - f_1(1420)$ mixing angle $\sim 50^\circ$ extracted from the analysis for a substantial body of data concerning the $f_1(1420)$ and $f_1(1285)$[15].

Within the nonrelativistic constituent quark model, the results regarding the masses of the $K_1(^1P_1)$ and $K_1(^3P_1)$, $(M_{K_1(^1P_1)}, M_{K_1(^3P_1)}) = (1368, 1306)$ MeV suggested by [8] and $(M_{K_1(^1P_1)}, M_{K_1(^3P_1)}) = (1356, 1322)$ MeV suggested by [9], are in good agreement with our predicted result. However, based on the following relation employed by [8, 9]

$$\tan^2(2\theta_K) = \left(\frac{M^2_{K_1(^3P_1)} - M^2_{K_1(^1P_1)}}{M^2_{K_1(1400)} - M^2_{K_1(1270)}}\right)^2 - 1, \tag{14}$$

the values of $\theta_K = (31 \pm 4)^\circ$ given by[8] and $\theta_K = (37.3 \pm 3.2)^\circ$ given by[9] disagree with value of $|\theta_K| \simeq (59.29 \pm 2.87)^\circ$ given by the present work.

As pointed out by our previous paper [16], (14) is equivalent to (12), and will yield two solutions $|\theta_K|$ and $\frac{\pi}{2} - |\theta_K|$. Simultaneously considering the relations (10), (11) and (14), in the presence of $M_{K_1(1400)} > M_{K_1(1270)}$, we can conclude that if $M_{K_1(^3P_1)} < M_{K_1(^1P_1)}$, the $|\theta_K|$ would greater than 45° . In fact, relation (12) clearly indicates that in the presence of $M_{K_1(1400)} > M_{K_1(1270)}$, the case $M_{K_1(^3P_1)} < M_{K_1(^1P_1)}$ must require $45^\circ < |\theta_K| < 90^\circ$.

3 The sign of θ_K constrained by experimental information

Now we wish to discuss the sign of θ_K by considering the open-flavor strong decays of the

$K_1(1270)$ and $K_1(1400)$ in the $^3P_0$ decay model, and the production ratio of these two physical strange states in the τ decay.

3.1 Strong decays of the $K_1(1270)$ and $K_1(1400)$ in the $^3P_0$ model

The main assumption of the $^3P_0$ decay model is that the strong decays take place via the production of a quark-antiquark pair with the vacuum quantum numbers which corresponds to the $^3P_0$ state of a quark-antiquark pair. After the $^3P_0$ decay model was originally introduced by Micu[21], it was applied extensively to meson and baryon decays. It is widely accepted that the $^3P_0$ model is successful since it gives a good description of many of the observed decay amplitudes and partial widths of the open-flavor meson strong decays.

Assuming a fixed $^3P_0$ source strength, simple harmonic oscillator quark model meson wave functions and physical phase space, Ackleh et al.[22] developed a diagrammatic, momentum-space formulation of the $^3P_0$ model to evaluate the partial width $\Gamma_{A\to BC}$

$$\Gamma_{A\to BC} = 2\pi \frac{PE_BE_C}{M_A}\sum_{LS}|\mathcal{M}_{LS}|^2, \tag{15}$$

where P is the decay momentum for the decay $A \to B + C$, E_B and E_C are the energies of mesons B and C, in the rest frame of A,

$$\begin{aligned} P &= \frac{[(M_A^2-(M_B+M_C)^2)(M_A^2-(M_B-M_C)^2)]^{1/2}}{2M_A}, \\ E_B &= (M_A^2 - M_C^2 + M_B^2)/2M_A, \\ E_C &= (M_A^2 - M_B^2 + M_C^2)/2M_A, \end{aligned}$$

M_A, M_B and M_C denote the masses of the mesons A, B and C, respectively; $\mathcal{M}_{LS}$ are proportional to an overall Gaussian in $x = P/\beta$ times a channel-dependent polynomial $\mathcal{P}_{LS}$,

$$\mathcal{M}_{LS} = \frac{\gamma}{\pi^{1/4}\beta^{1/2}}\mathcal{P}_{LS}(x)e^{-x^2/12}.$$

It is found that this formulation with the width parameter $\beta = 0.4$ GeV and the pair-production strength parameter $\gamma = 0.4$ can give a reasonably accurate description of the overall scale of decay widths[23, 24].

Based on (8) and (15), employing the analytical results for $\mathcal{P}_{LS}$ listed in Appendix A of Ref.[23], one can have[24]

$$\Gamma(K_1(1270) \to \rho K) = 21.8 \cos^2\theta_K + 61.6 \sin\theta_K \cos\theta_K + 43.6 \sin^2\theta_K, \tag{16}$$

$$\Gamma(K_1(1270) \to \pi K^*) = 59.6 \cos^2\theta_K - 158.7 \sin\theta_K \cos\theta_K + 115.7 \sin^2\theta_K, \tag{17}$$

$$\Gamma_{\text{thy}}(K_1(1270)) = 81 \cos^2\theta_K - 97 \sin\theta_K \cos\theta_K + 159 \sin^2\theta_K, \tag{18}$$

$$\Gamma(K_1(1400) \to \rho K) = 160 \cos^2\theta_K - 219.9 \sin\theta_K \cos\theta_K + 82.3 \sin^2\theta_K, \tag{19}$$

$$\Gamma(K_1(1400) \to \omega K) = 52.3 \cos^2\theta_K - 72.3 \sin\theta_K \cos\theta_K + 26.8 \sin^2\theta_K, \tag{20}$$

$$\Gamma(K_1(1400) \to \pi K^*) = 141.1 \cos^2\theta_K + 176.2 \sin\theta_K \cos\theta_K + 78.8 \sin^2\theta_K, \tag{21}$$

$$\Gamma_{\text{thy}}(K_1(1400)) = 353 \cos^2\theta_K - 116 \sin\theta_K \cos\theta_K + 188 \sin^2\theta_K, \tag{22}$$

$$|D/S|^2 = \begin{cases} \frac{(-0.0411\cos\theta_K - 0.029\sin\theta_K)^2}{(-0.204\cos\theta_K + 0.288\sin\theta_K)^2}, & \text{for } K_1(1270) \to \pi K^* \\ \frac{(-0.0498\cos\theta_K + 0.0704\sin\theta_K)^2}{(+0.247\cos\theta_K + 0.175\sin\theta_K)^2}, & \text{for } K_1(1400) \to \pi K^* \end{cases}. \tag{23}$$

For $\theta_K = \pm(59.29 \pm 2.87)^\circ$, the theoretical results regarding the above widths are shown in Tables 2, 3 and 4. Tables 2-4 clearly indicate that the present experimental data strongly prefer $\theta_K = +(59.29 \pm 2.87)^\circ$ over $\theta_K = -(59.29 \pm 2.87)^\circ$.

$K_1(1270)$	Exp.[19]	$\theta_K = +(59.29 \pm 2.87)^\circ$	$\theta_K = -(59.29 \pm 2.87)^\circ$
Γ (MeV)	**90±20**	96.07 ± 5.76	181.25 ± 1.11
$\Gamma(\rho K)/\Gamma(\pi K^*)$	**2.625±0.902**	2.07 ± 0.41	0.064 ± 0.014

Table 2: The predicted results of the $K_1(1270)$ strong decays in the $^3P_0$ decay model.

$K_1(1400)$	Exp.[19]	$\theta_K = +(59.29 \pm 2.87)^\circ$	$\theta_K = -(59.29 \pm 2.87)^\circ$
Γ (MeV)	**174±13**	180.1 ± 4.48	282.0 ± 10.0
$\Gamma(\rho K)/\Gamma$	**0.03±0.03**	0.033 ± 0.01	0.71 ± 0.04
$\Gamma(\omega K)/\Gamma$	**0.01±0.01**	0.0095 ± 0.0034	0.23 ± 0.01
$\Gamma(\pi K^*)/\Gamma$	**0.94±0.06**	0.96 ± 0.05	0.063 ± 0.006

Table 3: The predicted results of the $K_1(1400)$ strong decays in the $^3P_0$ decay model.

$\|D/S\|^2$	Exp.[19]	$\theta_K = +(59.29 \pm 2.87)^\circ$	$\theta_K = -(59.29 \pm 2.87)^\circ$
$K_1(1270) \to \pi K^*$	**1.0±0.7**	0.1 ± 0.03	0.0001 ± 0.0002
$K_1(1400) \to \pi K^*$	**0.04±0.01**	0.02 ± 0.004	12.5 ± 15.6

Table 4: The $|D/S|^2$ ratios for $K_1(1270) \to \pi K^*$ and $K_1(1400) \to \pi K^*$ in the $^3P_0$ model.

3.2 Production ratio of the $K_1(1270)$ and $K_1(1400)$ in the τ decay

With the definition of the decay constant of the axial-vector meson given by[5]

$$\langle 0|A_\mu|A(q,\varepsilon)\rangle = f_A m_A \varepsilon_\mu, \tag{24}$$

the partial width for $\tau \to \nu_\tau K_1$ can be expressed by

$$\Gamma(\tau \to \nu_\tau K_1) = \frac{G_F^2}{16\pi}|V_{us}|^2 f_{K_1}^2 \frac{(m_\tau^2 + 2m_{K_1}^2)(m_\tau^2 - m_{K_1}^2)^2}{m_\tau^3}. \tag{25}$$

Considering the SU(3) breaking corrections, following Ref.[5, 6], we have

$$\frac{f_{K_1(1270)} m_{K_1(1270)}}{f_{K_1(1400)} m_{K_1(1400)}} = \frac{\sin\theta_K - \delta\cos\theta_K}{\cos\theta_K + \delta\sin\theta_K}, \tag{26}$$

where the parameter δ denoting a SU(3) breaking factor has the following form in the static limit of the quark model[10][2]

$$\delta = \frac{1}{\sqrt{2}}\frac{m_s - m_u}{m_s + m_u} = 0.16 \pm 0.0003. \tag{27}$$

From (25) and (26), the $K_1(1400)$ and $K_1(1270)$ production ratio in the τ decay can be given by

$$\frac{\Gamma(\tau \to \nu_\tau K_1(1270))}{\Gamma(\tau \to \nu_\tau K_1(1400))} = F_p \left|\frac{\sin\theta_K - \delta\cos\theta_K}{\cos\theta_K + \delta\sin\theta_K}\right|^2, \tag{28}$$

where F_p denotes the phase factor given by

$$F_p = \frac{\left(m_\tau^2 + 2m_{K_1(1270)}^2\right)\left(m_\tau^2 - m_{K_1(1270)}^2\right)^2 m_{K_1(1400)}^2}{\left(m_\tau^2 + 2m_{K_1(1400)}^2\right)\left(m_\tau^2 - m_{K_1(1400)}^2\right)^2 m_{K_1(1270)}^2} = 1.82 \pm 0.086.$$

Then, from (28), one can have

$$\frac{\Gamma(\tau \to \nu_\tau K_1(1270))}{\Gamma(\tau \to \nu_\tau K_1(1400))} = \begin{cases} 2.62 \pm 0.55, & \text{for} \quad \theta_K = +(59.29 \pm 2.87)^\circ \\ 11.59 \pm 3.43, & \text{for} \quad \theta_K = -(59.29 \pm 2.87)^\circ \end{cases}. \tag{29}$$

[2] $m_u = 307.8 \pm 0.19$ MeV and $m_s = 488.69 \pm 0.28$ MeV derived from (1).

Experimentally, the $\mathcal{B}(\tau \to \nu_\tau K_1(1270))$ and $\mathcal{B}(\tau \to \nu_\tau K_1(1400))$ have been reported by TPC/Two-Gamma collaboration[25] in 1994 and ALEPH collaboration[26] in 1999, respectively. The averaged result of these two collaborations is given by[19]

$$\begin{aligned} \mathcal{B}(\tau \to \nu_\tau K_1(1270)) &= (0.47 \pm 0.11) \times 10^{-2} \\ \mathcal{B}(\tau \to \nu_\tau K_1(1400)) &= (0.17 \pm 0.26) \times 10^{-2} \end{aligned} , \tag{30}$$

which gives that

$$\left.\frac{\Gamma(\tau \to \nu_\tau K_1(1270))}{\Gamma(\tau \to \nu_\tau K_1(1400))}\right|_{\text{exp-1}} = 2.76 \pm 4.28. \tag{31}$$

This measured result also is in favor of $\theta_K = +(59.29 \pm 2.87)^\circ$ over $\theta_K = -(59.29 \pm 2.87)^\circ$, although the uncertainty of the reported result is large as shown in (31).

Assuming the resonance structure of $\tau^- \to K^-\pi^+\pi^-\nu_\tau$ decays being dominated by the $K_1(1270)$ and $K_1(1400)$ resonances, in 2000, both CLEO collaboration[4] and OPAL collaboration[27] have also measured the ratio of $\nu_\tau K_1(1270)$ to $\nu_\tau K_1(1400)$ with the averaged result[19]

$$\frac{\Gamma(\tau \to \nu_\tau K_1(1270))}{\Gamma(\tau \to \nu_\tau K_1(1270)) + \Gamma(\tau \to \nu_\tau K_1(1400))} = 0.69 \pm 0.15, \tag{32}$$

which therefore in turn implies that

$$\left.\frac{\Gamma(\tau \to \nu_\tau K_1(1270))}{\Gamma(\tau \to \nu_\tau K_1(1400))}\right|_{\text{exp-2}} = 2.23 \pm 1.56. \tag{33}$$

Comparison of (29) and (33) again shows that the present experimental data strongly prefer $\theta_K = +(59.29 \pm 2.87)^\circ$ over $\theta_K = -(59.29 \pm 2.87)^\circ$.

It is based on the $K_1(1400)$ production dominance in the τ decay that Suzuki suggests that the preferred result is $\theta_K \approx 33^\circ$ rather than 57°[6]. However, the recent available experiment data shown in (33) clearly show the $K_1(1270)$ dominance in the τ decay. Consequently, the argument of ruling out $\theta_K \approx 57^\circ$ from the $K_1(1400)$ dominance is therefore no longer valid. The study of hadronic decays $D \to K_1(1270)\pi, K_1(1400)\pi$ decays performed by Cheng[5] favors $\theta_K \approx -58^\circ$, however as pointed out by Cheng et al. in Ref.[28] that this argument is subject to many uncertainties such as the unknown $D \to K_1(^1P_1), K_1(^3P_1)$ transition form factors and the decay constants of $K_1(1270)$ and $K_1(1400)$. We note that the recent analysis for the SU(3) nonets of the axial vector mesons into a vector and a pseudoscalar performed by Roca et al.[29] based on a tensor formulation of the vector and axial vector fields gives $\theta_K = +(62 \pm 3)^\circ$, which is in fact in good agreement with our suggested result that $\theta_K = +(59.29 \pm 2.87)^\circ$.

4 Concluding remarks

In the nonrelativistic constituent quark model, the masses of the $K_1(^3P_1)$ and $K_1(^1P_1)$ are determined to be 1307.88 ± 10.33 and 1396.5 ± 1.92 MeV, respectively, which therefore suggests that the absolute value of the $K_1(^3P_1) - K_1(^1P_1)$ mixing angle is $(59.29 \pm 2.87)^\circ$. These findings are in good agreement with those given by Ref.[16] based on the investigation on the implication of the $f_1(1285) - f_1(1420)$ mixing for the $K_1(^3P_1) - K_1(^1P_1)$ mixing angle. Investigating the open-flavor strong decays of the $K_1(1270)$ and $K_1(1400)$ in the $^3P_0$ decay model, we find the current experimental data strongly prefer $\theta_K = +(59.29 \pm 2.87)^\circ$ over $\theta_K = -(59.29 \pm 2.87)^\circ$. The analysis for the production ratio of the $K_1(1270)$ and $K_1(1400)$ in the τ decay also indicates that the experimental data is in favor of the result $\theta_K = +(59.29 \pm 2.87)^\circ$.

In the framework of a covariant light-front quark model, the calculations performed by Cheng et al.[28] for the exclusive radiative B decays, $B \to K_1(1270)\gamma$, $K_1(1400)\gamma$, show that the relative strength of $B \to K_1(1270)\gamma$ and $B \to K_1(1270)\gamma$ rates is very sensitive to the sign of the θ_K. The recent analysis of two-body B decays with an axial-vector meson in the final state performed by Nardulli et al.[30, 31] using naive factorization, shows the branching ratios for $B \to b_1\pi$, b_1K, $a_1\pi$ and a_1K also depend strongly on the θ_K. In addition, as pointed by Suzuki[32], the relation $|Am(J/\psi(\psi') \to K_1^0(1400)\overline{K^0}|^2 = \tan^2\theta_K|Am(J/\psi(\psi') \to K_1^0(1270)\overline{K^0}|^2$ can be able to determine the θ_K directly without referring to other parameters. Therefore, in order to further check the consistency of our suggested mixing angle of $K_1(1270)$ and $K_1(1400)$, detailed experimental study of the above mentioned decays involving the axial-vector mesons is certainly desirable.

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